Sovereign Defaults

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1 Motivation

Two recent papers advance frontiers of sovereign default modeling. First, Aguiar and Gopinath (2006) highlight the importance of fluctuations in long-term productivity growth for sovereign default dynamics in emerging markets. Lately, Mendoza and Yue (2012) build a quantitative model in which both sovereign default spreads and output costs of default are determined endogenously. The output costs of default arise because domestic producers loose access to trade credit and are forced to substitute away from foreign intermediate inputs. Both papers substantially increase debt-to-GDP ratios of defaulting countries, relative to those attained in earlier sovereign default models. Nevertheless, debt-to-GDP ratios in both papers still fall short of those ratios observed in the data (23 percent in the models, versus 35 to 71 percent in the data). Both papers are calibrated to Argentinean data and have little to say about sovereign defaults in developed countries.

In this project we investigate sovereign defaults in the presence of temporary and permanent shocks to firms' total factor productivity. We introduce fluctuations in trend growth to the model of Mendoza and Yue (2012) to achieve two objectives. First, we want to improve the model's fit of debt-to-GDP ratios in the data. Second, the model will account for default spread dynamics in both emerging and developed economies by allowing for differences in trend growth volatility, as measured by Aguiar and Gopinath (2007).

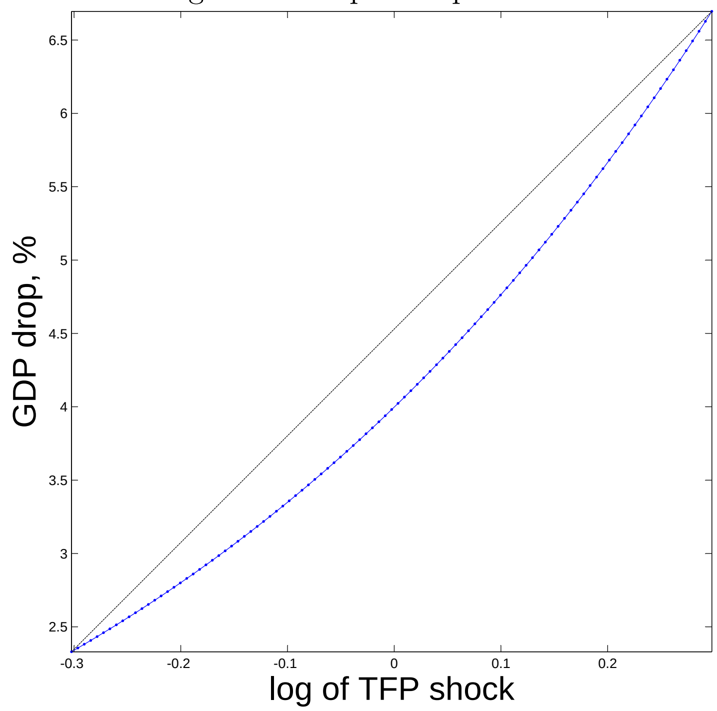
2 Two Models with Stable Trend and Growth Schocks

We are considering two extensions of the baseline model of Mendoza and Yue (2012). In addition to transitory TFP shocks, models 1 and 2 introduce stable and volatile trend to TFP respectively.

There are four agents: households, firms, sovereign government and risk neutral foreign lenders. There are two sectors: final f and intermediate m goods. Final goods producers borrow working capital at fixed interest rate r^* from abroad to pay for subset of imported intermediate inputs.

Sovereign default excludes both firms and government from world credit markets for a period of time as a punishment in a standard framework of Eaton and Gersovitz (1981). Default causes final good firms to incur efficiency loss due to imperfect substitution of imported intermediate goods. Default cost (output drop) is an increasing convex function of

Figure 1: Output drop in default



TFP determined endogenously. Figure 1 depicts the drop in GDP at the same period of default as a function of the TFP shock.

The TFP process ε consists of two components: transitory shock z and permanent shock Γ. Both models add permanent growth to transitory z TFP shocks ε through

$$\begin{aligned} \varepsilon_t &= \exp(z_t + \Gamma_t) \\ z_t &= \mu_z(1-\rho_z) + \rho_z z_{t-1} + \varepsilon_{zt} \\ \Gamma_t &= \mu_\Gamma + \Gamma_{t-1} + g_t \\ g_t &= \mu_g(1-\rho_g) + \rho_g g_{t-1} + \varepsilon_{gt} \end{aligned}$$

where ε_{zt} and ε_{gt} are i.i.d. normal random variables with zero mean and standard deviations σ_z and σ_g respectively. Define

$$\hat{\varepsilon}_t = \varepsilon_t \exp(\mu_\Gamma + \Gamma_{t-1}) = \exp(z_t + g_t)$$

Then in order for $E[\hat{\varepsilon}] = 1$ to hold, we need to set $\mu_z = -\frac{\sigma_z^2}{2(1-\rho_z^2)}$ and $\mu_g = -\frac{\sigma_g^2}{2(1-\rho_g^2)}$. In model 1, growth is stable with $\sigma_g = 0$. Model 2 features additional shock g to permanent growth μ_Γ by allowing $\sigma_g > 0$.

For labour hours to remain stationary in household's problem, we need to use Cobb-Douglas preferences or adjust GHH preferences with permanent productivity shocks on disutility of labor:

$$\begin{aligned} \max_{c_t, L_t} \quad & E_0 \sum_{t=0}^{\infty} \frac{\beta^t}{1-\sigma} \left(c_t - \exp(\mu_\Gamma + \Gamma_{t-1}) \frac{L_t^\omega}{\omega} \right)^{1-\sigma} \\ \text{s.t.} \quad & c_t = w_t L_t + \pi_t^f + \pi_t^m + T_t \end{aligned}$$

where w_t is the wage rate, π_t^f, π_t^m are profits paid by firms in f and m sectors, and T_t is government transfers.

Detrending by $\exp(\mu_\Gamma + \Gamma_{t-1})$ yields

$$\begin{aligned} \max_{\hat{c}_t, L_t} \quad & E_0 \sum_{t=0}^{\infty} \frac{\prod_{\tau=0}^{t} \hat{\beta}_\tau}{1-\sigma} \left(\hat{c}_t - \frac{L_t^\omega}{\omega} \right)^{1-\sigma} \\ \text{s.t.} \quad & \hat{c}_t = \hat{w}_t L_t + \hat{\pi}_t^f + \hat{\pi}_t^m + \hat{T}_t \end{aligned}$$

where $\hat{\beta}_t = \beta \exp[(1-\sigma)(\mu_\Gamma + g_{t-1})]$.

The firm's production function is Cobb-Douglas with CES Armington aggregator of intermediate goods. Final goods producers solve

$$\begin{aligned} \max_{L_t^f, m_t^d, m_t^*} \quad & \hat{\pi}_t^f = \hat{y}_t - \hat{w}_t L_t^f - \hat{p}_t^m m_t^d - \hat{p}_t^* m_t^* \\ \text{s.t.} \quad & \hat{y}_t = \hat{\varepsilon}_t \big(M(m_t^d, m_t^*) \big)^{\alpha_M} \big(L_t^f \big)^{\alpha_L} \\ & M(m_t^d, m_t^*) = \big[\lambda (m_t^d)^\eta + (1-\lambda)(m_t^*)^\eta \big]^{\frac{1}{\eta}} \\ & \hat{\varepsilon}_t = \exp(z_t + g_t) \end{aligned}$$

where m_t^* is Dixit-Stiglitz aggregator of continuum of differentiated imported inputs $m_j^*, j \in [0,1]$. Subset θ of imported inputs requires final goods producers to obtain working capital financing at interest rate r_t^*:

$$\begin{aligned} \max_{\{m_{jt}^*\}_{j=\in[0,1]}} \quad & \hat{p}_t^* m_t^* - \int_0^1 \hat{q}_j^* m_{jt}^* dj - r_t^* \int_0^\theta \hat{q}_j^* m_{jt}^* dj \\ \text{s.t} \quad & m_t^* = \left[\int_0^1 (m_{jt}^*)^\nu dj \right]^{\frac{1}{\nu}} \end{aligned}$$

Normalizing prices of differentiated imported inputs $\hat{q}_j^* = 1, \forall j$, we obtain the aggregate price of imported inputs as a function of interest rate:

$$\hat{p}_t^* = [\theta(1+r_t^*)^{\frac{\nu}{\nu-1}} + 1 - \theta]^{\frac{\nu-1}{\nu}}$$

In default, when $r_t^* = \infty$, we obtain $\hat{p}_t^* = (1-\theta)^{\frac{\nu-1}{\nu}}$. GDP is defined as $\widehat{gdp}_t = \hat{y}_t - \hat{p}_t^* m_t^*$. Intermediate goods producers solve

$$\begin{aligned} \max_{L_t^m} \quad & \hat{\pi}_t^m = \hat{p}_t^m m_t^d - \hat{w}_t L_t^m \\ \text{s.t} \quad & m_t^d = A(L_t^m)^\gamma \end{aligned}$$

Dynamic Problem of Sovereign Government is identical to Eaton and Gersovitz (1981) with endogenous link between sovereign default and private economics activity.

$$\begin{aligned}
\hat{V}(\hat{a}_t, \hat{\varepsilon}_t) &= \max\Big\{\hat{V}^G(\hat{a}_t, \hat{\varepsilon}_t), \hat{V}^B(\hat{\varepsilon}_t)\Big\} \\
\hat{V}^G(\hat{a}_t, \hat{\varepsilon}_t) &= \max_{\hat{c}_t, \hat{a}_{t+1}} U\Big(\hat{c}_t - \frac{L_t^\omega}{\omega}\Big) + \hat{\beta}_t E\big[\hat{V}(\hat{a}_{t+1}, \hat{\varepsilon}_{t+1})\big] \\
&\quad \text{s.t.} \quad \hat{c}_t = \hat{y}_t - \hat{p}_t^* m_t^* + \hat{a}_t - \exp(\mu_\Gamma + g_t)\hat{q}(\hat{a}_{t+1}, \hat{\varepsilon}_t)\hat{a}_{t+1} \\
\hat{V}^B(\hat{\varepsilon}_t) &= \max_{\hat{c}_t} U\Big(\hat{c}_t - \frac{L_t^\omega}{\omega}\Big) + \hat{\beta}_t E\big[\phi\hat{V}(0, \hat{\varepsilon}_{t+1}) + (1-\phi)\hat{V}^B(\hat{\varepsilon}_{t+1})\big] \\
&\quad \text{s.t.} \quad \hat{c}_t = \hat{y}_t - \hat{p}_t^* m_t^* \\
\hat{q}(\hat{a}_{t+1}, \hat{\varepsilon}_t) &= \frac{1}{1+r_t^*} Pr\big(\hat{V}^G(\hat{a}_{t+1}, \hat{\varepsilon}_{t+1}) > \hat{V}^B(\hat{\varepsilon}_{t+1})\big)
\end{aligned}$$

3 Solution Algorithm

State space consists of credit history h, debt $a > 0$, and TFP ε. Algorithm is very similar to the rest of the quantitative literature on sovereign debt, except we solve for the recursive equilibrium in two steps:

1. Static problem of private sector is solved first to determine equilibrium production plans: factor allocations, domestic and imported intermediate inputs, prices and final output as functions of TFP only.

2. Dynamic problem of sovereign government is solved for the optimal default decisions in debt market equilibrium.

The static problem of private sector and dynamic problem of government can be solved separately because

1. GHH preferences allow pro-cyclical labor hours and remove the wealth effect on labor supply by making the marginal rate of substitution between consumption and labor depend on labor only: $\hat{w}_t = L_t^{\omega-1}$. This allows production plans depend on TFP only. In case of Cobb-Douglas utility these would be functions of two state variables: TFP and debt.

2. Global interest rate on working capital loans is fixed. As an extension we could model foreign lenders risk averse to create upward sloping interest rate schedule on working capital loans.

If either of these two assumptions were relaxed, dynamic problem would have to be solved simultaneously with static problem.

4 Calibration

Calibration is identical to Mendoza and Yue (2012).

$\alpha_M = 0.43$	$\alpha_k = 0.17$	$\alpha_L = 0.40$
$\beta = 0.88$	$\sigma = 2$	$\omega = 1.455$
$r^* = 1\%$	$\nu = 0.59$	$\theta = 0.7$
$A = 0.31$	$\gamma = 0.7$	$\phi = 0.083$
$\lambda = 0.62$	$\eta = 0.65$	

In addition to transitory TFP shocks, models 1 and 2 introduce stable and volatile trend to TFP respectively:

Model 1:	$\rho_z = 0.95$	$\sigma_z = 1.7\%$	$\rho_g = 0$	$\sigma_g = 0$	$\mu_\Gamma = 2\%$
Model 2:	$\rho_z = 0$	$\sigma_z = 0\%$	$\rho_g = 0$	$\sigma_g = 1.7\%$	$\mu_\Gamma = 2\%$

5 Results

Table 1 summarizes the results of a sensitivity analysis of model 1 to changes in the growth rate of TFP μ_Γ. It evaluates the robustness of model 1 predictions of debt to GDP ratio and quarterly default frequency. Row (1) reports the statistics from the Argentine data. Rows (2)-(5) report results varying the value of μ_Γ. Row (2) removes TFP trend altogether by setting $\mu_\Gamma = 0$, whereas rows(3), (4) and (5) set μ_Γ to 1%, 2% and 3% respectively. Without TFP trend, the model is identical to baseline Mendoza and Yue (2012). Adding stable quarterly growth rate to TFP increases debt to GDP ratio to 33% and matches the debt ratio from the data while *at the same time* keeping the 0.14% default frequency. Row (6) reports that replacing persistent transitory shocks by i.i.d. trend shocks with the same standard deviation 1.7% further increases the debt ratio to 39%, but lowers default frequency to .10%. Simulation statistics in table 2 illustrates improved correlation of macro variables with output.

References

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Table 1: Sensitivity Analysis

	Output drop at default	Mean debt/GDP ratio	Default frequency	Mean spread	Std. dev. of spread	GDP corr. with spread	GDP corr. with default
(1) Data	13%	35%	0.69%	1.86%	0.78%	-0.62	-0.11
Model 1							
(2) $\mu_\Gamma = 0\%$	14.3%	21.9%	0.13%	0.57%	0.29%	-0.31	-0.11
(3) $\mu_\Gamma = 1\%$	14.3%	25.8%	0.14%	0.59%	0.29%	-0.27	-0.11
(4) $\mu_\Gamma = 2\%$	14.4%	32.7%	0.14%	0.57%	0.27%	-0.23	-0.10
(5) $\mu_\Gamma = 3\%$	15.2%	61.3%	0.07%	0.27%	0.12%	-0.03	-0.11
(6) Model 2	14.1%	39.1%	0.10%	0.42%	0.21%	-0.61	-0.17

Table 2: Simulation Statistics

Statistics	Data	Model 1	Model 2
Average debt/GDP ratio	35%	32.7%	39.1%
Average bond spreads	1.86%	0.57%	0.42%
GDP autocorrelation	0.82	0.71	0.29
Std. dev. of GDP	4.7%	4.53%	3.26%
Std. dev. of trade balance		1.10%	0.84%
Std. dev. of bond spreads	0.78%	0.27%	0.21%
Consumption std.dev./GDP std.dev.	1.44	1.10	1.15
Correlations with GDP			
bond spreads	-0.62	-0.23	-0.61
trade balance	-0.87	-0.34	-0.63
labor	0.39	0.55	0.74
intermediate goods	0.90	0.96	0.74
Correlations with bond spreads			
trade balance	0.82	0.37	0.58
labor	-0.42	-0.18	-0.77
intermediate goods	-0.39	-0.23	-0.72
Historical default-output co-movements			
correlation between default and GDP	-0.11	-0.11	-0.17
frac.of def. with GDP below trend	61.5%	99.5%	100.0%
frac.of def. with large recessions	32.0%	50.7%	97.0%
Default frequency	0.69%	0.13%	0.10%
Output drop in default	13%	14.3%	14.1%

Figure 2: Solution to optimal default problem

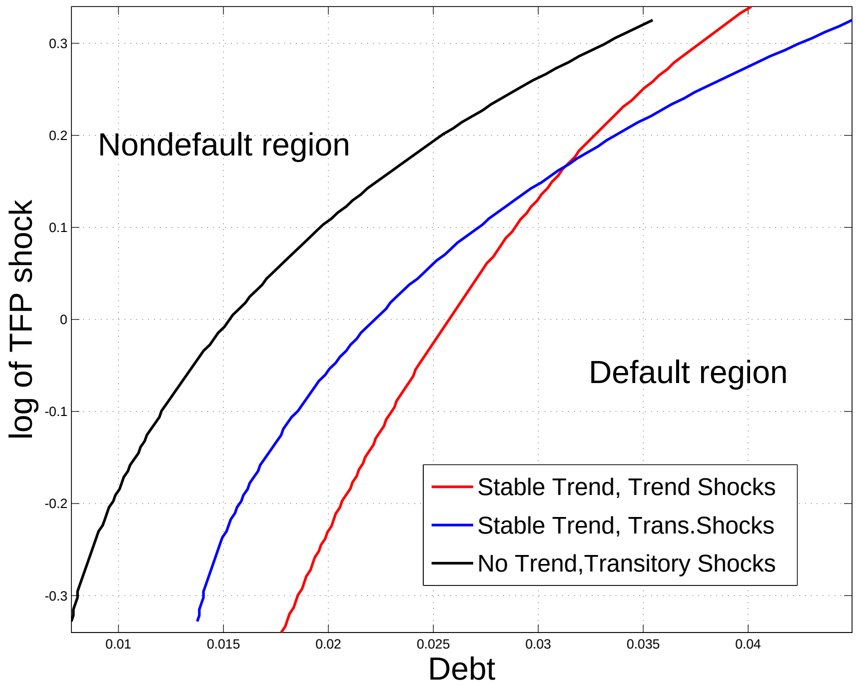

Figure 3: Macroeconomic dynamics around default episodes