Model of sovereign defaults with growth shocks

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Motivation

Two recent papers advance frontiers of sovereign default modeling:

1. Mendoza and Yue (QJE 2012) build a quantitative model in which both sovereign default spreads and output costs of default are determined endogenously. The output costs of default arise because domestic producers loose access to trade credit and are forced to substitute away from foreign intermediate inputs.
2. Aguiar and Gopinath (JIE 2006) highlight the importance of fluctuations in long-term productivity growth for sovereign default dynamics in emerging markets.

Both papers substantially increase debt-to-GDP ratios of defaulting countries, relative to those attained in earlier sovereign default models. Nevertheless, debt-to-GDP ratios in both papers still fall far short of those ratios observed in the data.

This project

In this project we would like to introduce fluctuations in trend growth to the model of Mendoza and Yue, to achieve two objectives:

1. improve the model's fit of debt-to-GDP ratios in the data
2. account for default spread dynamics in both emerging and developed economies by allowing for differences in trend growth volatility, as measured by Aguiar and Gopinath (JPE 2007)

Model Structure

- Four agents: households, firms, sovereign government and risk neutral foreign lenders
- Two sectors: final f and intermediate m goods
- Two shocks: transitory z and permanent growth g shocks that affect TFP ε through

$$\begin{aligned}
\varepsilon' &= \exp(z' + \Gamma') \\
z' &= \mu_z(1-\rho_z) + \rho_z z + \varepsilon'_z \\
\Gamma' &= \mu_\Gamma + \Gamma + g' \\
g' &= \mu_g(1-\rho_g) + \rho_g g + \varepsilon'_g
\end{aligned}$$

Model Structure

- Final goods producers borrow working capital at fixed interest rate r^* from abroad to pay for subset of imported intermediate inputs.
- Sovereign default excludes both firms and government from world credit markets for a period of time as a punishment
- Default causes final good firms to incurs efficiency loss due to imperfect substitution of imported intermediate goods.
- Default cost (output drop) is an **increasing convex function of TFP** determined endogenously.
- State space: credit history h, debt $a > 0$, TFP ε

Algorithm

Algorithm is very similar to the rest of the quantitative literature on sovereign debt, except we solve for the recursive equilibrium in two steps:

1. Static problem of private sector is solved first to determine equilibrium production plans: factor allocations, domestic and imported intermediate inputs, prices and final output as functions of the state variables.
2. Dynamic problem of sovereign government is solved for the optimal default decisions in debt market equilibrium.

Static Problem: Households

For labour hours to remain stationary, we need to use Cobb-Douglas preferences or adjust GHH preferences with permanent productivity shocks on disutility of labor:

$$\max_{c_t, L_t} \quad E_0 \sum_{t=0}^{\infty} \frac{\beta^t}{1-\sigma} \left(c_t - \exp(\mu_\Gamma + \Gamma_{t-1}) \frac{L_t^\omega}{\omega} \right)^{1-\sigma}$$

$$\text{s.t.} \quad c_t = w_t L_t + \pi_t^f + \pi_t^m + T_t$$

where w_t is the wage rate, π_t^f, π_t^m are profits paid by firms in f and m sectors, and T_t is government transfers.

Private debt contracts are not allowed.

Detrending

Detrending by $\exp(\mu_\Gamma + \Gamma_{t-1})$ yields

$$\max_{\hat{c}_t, L_t} \quad E_0 \sum_{t=0}^{\infty} \frac{\prod_{\tau=0}^{t} \hat{\beta}_\tau}{1-\sigma} \left(\hat{c}_t - \frac{L_t^\omega}{\omega} \right)^{1-\sigma}$$

$$\text{s.t.} \quad \hat{c}_t = \hat{w}_t L_t + \hat{\pi}_t^f + \hat{\pi}_t^m + \hat{T}_t$$

where $\hat{\beta}_t = \beta \exp[(1-\sigma)(\mu_\Gamma + g_t)]$.

Optimal condition $\hat{w}_t = L_t^{\omega-1}$ allows production plans depend on TFP only and separate static and dynamic problems.

Firms

- Final Goods Producers

$$
\begin{aligned}
\max_{L_t^f, m_t^d, m_t^*} \quad & \hat{\pi}_t^f = \hat{y}_t - \hat{w}_t L_t^f - \hat{p}_t^m m_t^d - \hat{p}_t^* m_t^* \\
\text{s.t.} \quad & \hat{y}_t = \hat{\varepsilon}_t \big(M(m_t^d, m_t^*)\big)^{\alpha_M} \left(L_t^f\right)^{\alpha_L} \\
& M(m_t^d, m_t^*) = \left[\lambda (m_t^d)^{\eta} + (1-\lambda)(m_t^*)^{\eta}\right]^{\frac{1}{\eta}} \\
& \hat{\varepsilon}_t = \exp(z_t + g_t)
\end{aligned}
$$

- Intermediate Goods Producers

$$
\begin{aligned}
\max_{L_t^m} \quad & \hat{\pi}_t^m = \hat{p}_t^m m_t^d - \hat{w}_t L_t^m \\
\text{s.t} \quad & m_t^d = A(L_t^m)^{\gamma}
\end{aligned}
$$

Imported Intermediate Inputs

- m_t^* is Dixit-Stiglitz aggregator of continuum of differentiated imported inputs $m_j^*, j \in [0,1]$.
- Subset θ of imported inputs requires final goods producers to obtain working capital financing at interest rate r_t^*:

$$\max_{\{m_{jt}^*\}_{j=\in[0,1]}} \quad \hat{p}_t^* m_t^* - \int_0^1 \hat{p}_j^* m_{jt}^* dj - r_t^* \int_0^\theta \hat{p}_j^* m_{jt}^* dj$$

$$\text{s.t} \quad m_t^* = \left[\int_0^1 (m_{jt}^*)^\nu dj\right]^{\frac{1}{\nu}}$$

- In default, $r_t^* = \infty$. Assuming $\hat{p}_j^* = 1, \forall j$, we get

$$\hat{p}_t^* = [\theta(1+r_t^*)^{\frac{\nu}{\nu-1}} + 1 - \theta]^{\frac{\nu-1}{\nu}} = (1-\theta)^{\frac{\nu-1}{\nu}}$$

- $\widehat{gdp}_t = \hat{y}_t - \hat{p}_t^* m_t^*$

Dynamic Problem: Sovereign Government

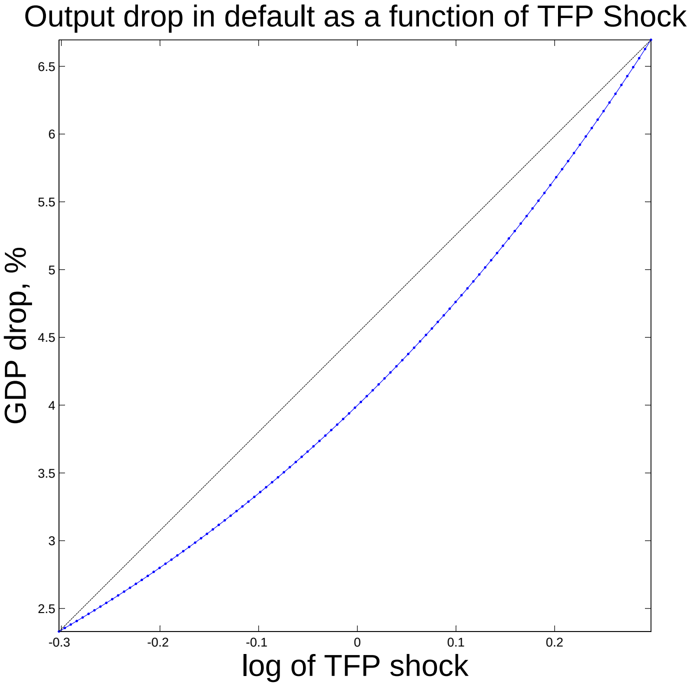
$$
\begin{aligned}
\hat{V}(\hat{a}_t, \hat{\varepsilon}_t) &= \max\Big\{\hat{V}^G(\hat{a}_t, \hat{\varepsilon}_t), \hat{V}^B(\hat{\varepsilon}_t)\Big\} \\
\hat{V}^G(\hat{a}_t, \hat{\varepsilon}_t) &= \max_{\hat{c}_t, \hat{a}_{t+1}} U\Big(\hat{c}_t - \frac{L_t^\omega}{\omega}\Big) + \hat{\beta}_t E\big[\hat{V}(\hat{a}_{t+1}, \hat{\varepsilon}_{t+1})\big] \\
&\text{s.t.} \quad \hat{c}_t = \hat{y}_t - \hat{p}_t^* m_t^* + \hat{a}_t - \exp(\mu_\Gamma + g_t)\hat{q}(\hat{a}_{t+1}, \hat{\varepsilon}_t)\hat{a}_{t+1} \\
\hat{V}^B(\hat{\varepsilon}_t) &= \max_{\hat{c}_t} U\Big(\hat{c}_t - \frac{L_t^\omega}{\omega}\Big) + \hat{\beta}_t E\big[\phi \hat{V}(0, \hat{\varepsilon}_{t+1}) + (1-\phi)\hat{V}^B(\hat{\varepsilon}_{t+1})\big] \\
&\text{s.t.} \quad \hat{c}_t = \hat{y}_t - \hat{p}_t^* m_t^* \\
\hat{q}(\hat{a}_{t+1}, \hat{\varepsilon}_t) &= \frac{1}{1 + r_t^*} Pr\big(\hat{V}^G(\hat{a}_{t+1}, \hat{\varepsilon}_{t+1}) > \hat{V}^B(\hat{\varepsilon}_{t+1})\big)
\end{aligned}
$$

Calibration

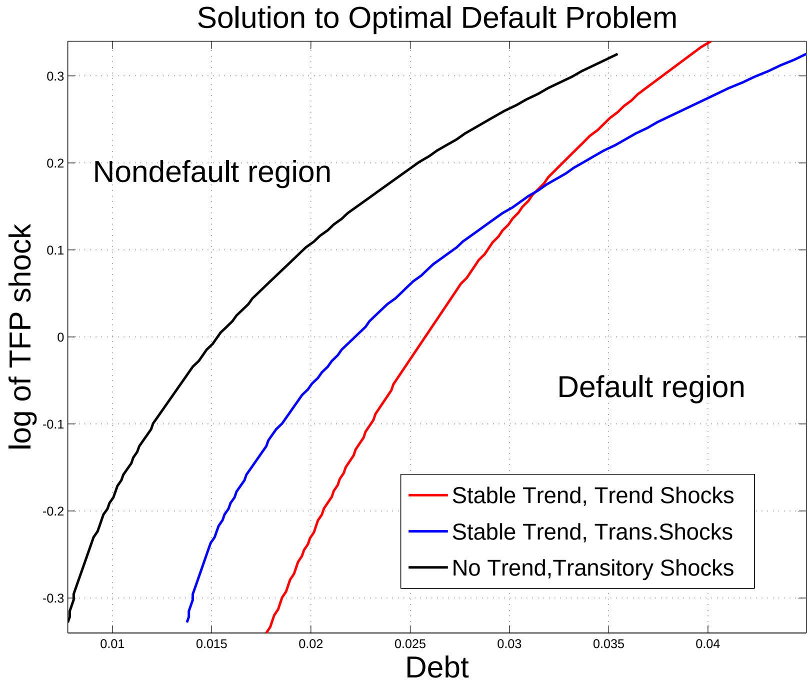
$\alpha_M = 0.43$	$\alpha_k = 0.17$	$\alpha_L = 0.40$
$\beta = 0.88$	$\sigma = 2$	$\omega = 1.455$
$r^* = 1\%$	$\nu = 0.59$	$\theta = 0.7$
$A = 0.31$	$\gamma = 0.7$	$\phi = 0.083$
$\lambda = 0.62$	$\eta = 0.65$	$\mu_\Gamma = 2\%$

Model 1:	$\rho_z = 0.9$	$\sigma_z = 1.7\%$	$\rho_g = 0$	$\sigma_g = 0$
Model 2:	$\rho_z = 0$	$\sigma_z = 0$	$\rho_g = 0$	$\sigma_g = 1.7\%$

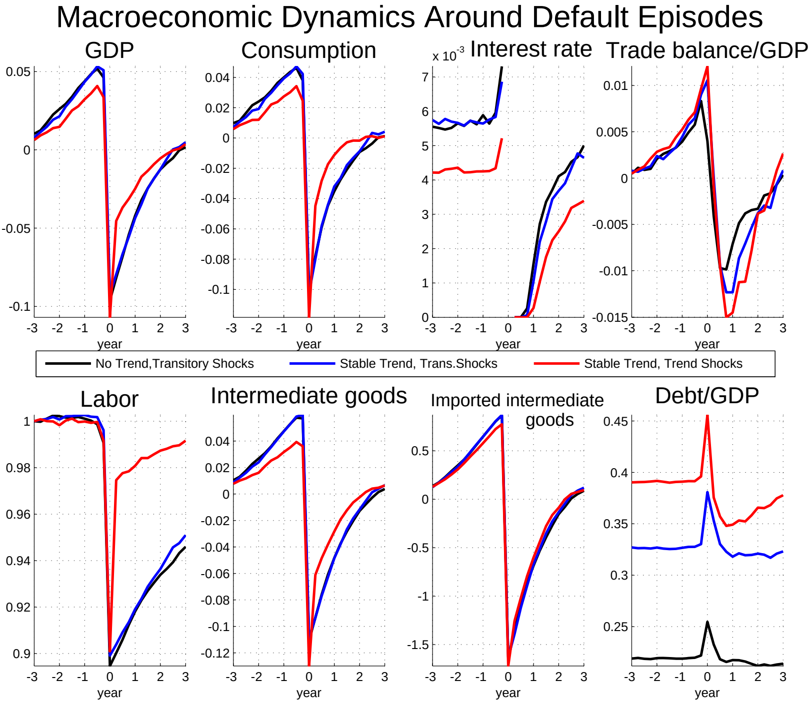
For $E[\hat{\varepsilon}] = 1$, $\mu_z = -\frac{\sigma_z^2}{2(1-\rho_z^2)}$ and $\mu_g = -\frac{\sigma_g^2}{2(1-\rho_g^2)}$.



Output drop in default as a function of TFP Shock
GDP drop, %
6.5
6
5.5
5
4.5
4
3.5
3
2.5
-0.3
-0.2
-0.1
0
0.1
0.2
log of TFP shock



Solution to Optimal Default Problem
log of TFP shock
Debt
Nondefault region
Default region
Stable Trend, Trend Shocks
Stable Trend, Trans.Shocks
No Trend,Transitory Shocks
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
0.01
0.015
0.02
0.025
0.03
0.035
0.04


Macroeconomic Dynamics Around Default Episodes
GDP
Consumption
x 10^-3 Interest rate
Trade balance/GDP
year
No Trend,Transitory Shocks
Stable Trend, Trans.Shocks
Stable Trend, Trend Shocks
Labor
Intermediate goods
Imported intermediate goods
Debt/GDP

Simulation Statistics

Statistics	Data	Model 1	Model 2
Average debt/GDP ratio	35%	32.7%	39.1%
Average bond spreads	1.86%	0.57%	0.42%
GDP autocorrelation	0.82	0.71	0.29
Std. dev. of GDP	4.7%	4.53%	3.26%
Std. dev. of trade balance		1.10%	0.84%
Std. dev. of bond spreads	0.78%	0.27%	0.21%
Consumption std.dev./GDP std.dev.	1.44	1.10	1.15
Correlations with GDP			
bond spreads	-0.62	-0.23	-0.61
trade balance	-0.87	-0.34	-0.63
labor	0.39	0.55	0.74
intermediate goods	0.90	0.96	0.74
Correlations with bond spreads			
trade balance	0.82	0.37	0.58
labor	-0.42	-0.18	-0.77
intermediate goods	-0.39	-0.23	-0.72

Simulation Statistics

	Data	Model 1	Model 2
Historical default-output co-movements			
correlation between default and GDP	-0.11	-0.11	-0.17
frac.of def. with GDP below trend	61.5%	99.5%	100.0%
frac.of def. with large recessions	32.0%	50.7%	97.0%
Default frequency	0.69%	0.13%	0.10%
Output drop in default	13%	14.3%	14.1%

Results

- Adding stable quarterly 2% growth rate to TFP increases debt to GDP ratio to 33% and matches the debt ratio from the data while *at the same time* keeping the 0.13% default frequency.
- Replacing persistent transitory shocks by i.i.d. trend shocks with the same standard deviation 1.7% further increases the debt ratio to 39%, but lowers default frequency to .10%.